APPENDIX

Equation (1) can be used to express the impedance combinations by (2):

$$Z_a(1) + Z_b(1) = \frac{1+k}{1-k} \tag{A1a}$$

$$Z_a(1) + Z_b(2) = \frac{1-k}{1+k} \tag{A1b}$$

$$Z_a(2) + Z_b(1) = \frac{1+jk}{1-jk} \tag{A1c}$$

$$Z_a(2) + Z_b(2) = \frac{1-jk}{1+jk}. \tag{A1d}$$

If one subtracts (A1a) from (A1b) and (A1c) from (A1d) and equals the results, a quadratic equation for k is obtained:

$$1 + k^2 = j(1 - k^2). \tag{A2}$$

The solution is (2).

To take the losses into account we now assume that the impedances Z_a and Z_b have small real parts:

$$Z_a(n) = r_a(n) + jX_a(n) \qquad Z_b(n) = r_b(n) + jX_b(n). \tag{A3}$$

The imaginary parts obey (1). Using the definition of $\hat{Q}(3)$ we can express the real parts by $\hat{Q}$ and the impedance change

$$r_a(1)\cdot r_a(2) = \frac{|Z_a(1) - Z_a(2)|^2}{\hat{Q}^2}. \tag{A4}$$

With the help of (A1), one can express the difference in the imaginary parts by k

$$Z_a(1) - Z_a(2) = r_a(1) - r_a(2) + \left(\frac{1+k}{1-k} - \frac{1+jk}{1-jk}\right). \tag{A5}$$

The square of the absolute value of this expression is then

$$|Z_a(1) - Z_a(2)|^2 = (r_a(1) - r_a(2))^2 + 4 \tag{A6}$$

where $k = \sqrt{j}$ was used. Since we have assumed $r_a(n)$ to be small, we neglect the difference $(r_a(1) - r_a(2))^2$ in (A6) and transform (A4) into the result (4). The product $r_b(1)\cdot r_b(2)$ in (4) can be determined in the same fashion.

We will now determine the absolute value of the reflection coefficient ρ_{11} corresponding to the two impedances $Z_a(1)$ and $Z_b(1)$. We abbreviate $\sigma = r_a(1) - r_b(1)$ and $\xi = X_a(1) - X_b(1)$. $|\rho_{11}|^2$ is then given by

$$|\rho_{11}|^2 = \frac{(\sigma - 1)^2 + \xi^2}{(\sigma + 1)^2 + \xi^2}. \tag{A7}$$

If one takes into account the smallness of r, neglecting all but the linear terms in r, (A7) transforms:

$$|\rho_{11}|^2 = 1 - \frac{4\sigma}{1 + \xi^2} \tag{A8a}$$

or

$$|\rho_{11}| = 1 - \frac{2\sigma}{1 + \xi^2}. \tag{A8b}$$

From (A1), $\xi^2 = (X_a(1) - X_b(1))^2$ can be determined to be $\xi^2 = (1 + \sqrt{2})^2$, which finally yields the result given in (5). The other reflection coefficient is obtained the same way.

To take into account the effects of the parallel resistor shown in Fig. 4, we first split the impedance of the parallel combination in real and imaginary parts:

$$Z = \frac{R(r + jx)}{R + r + jx} = \frac{R(x^2 + r(R + r))}{(R + r)^2 + x^2} + jx\frac{R^2}{x^2 + R^2} \tag{A9}$$

when $r = r_a + r_b$ and $x = x_a + x_b$. Since r is small and R very big compared to x, one can simplify

$$Z \approx r + \frac{x^2}{R} + jx. \tag{A10}$$

Inserting the modified real parts into (5) yields the magnitudes of the reflection coefficients:

$$|\rho_{11}| = |\rho_{22}| = 1 - \left(1 - \frac{1}{\sqrt{2}}\right)\cdot\left(r_a(1) + r_b(1) + \frac{1}{R}(X_a(1) + X_b(1))^2\right)$$

$$|\rho_{12}| = |\rho_{21}| = 1 - \left(1 + \frac{1}{\sqrt{2}}\right)\cdot\left(r_a(1) + r_b(1) + \frac{1}{R}(X_a(1) + X_b(2))^2\right). \tag{A11}$$

Equality of the absolute values of the reflection coefficients leads to a requirement for the resistance R. If one uses (A1) and (2), R assumes the value

$$R = \frac{1}{r_a(1) + r_b(1)}.$$

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Self-Oscillating Tunnel-Diode Mixer Having Conversion Gain

CANAN TOKER

***Abstract*—A self-oscillating-type tunnel-diode mixer having conversion gain operating at a signal frequency of 600 MHz is demonstrated. By changing the bias, conversion gains up to infinity (oscillations) can be obtained, although gains in excess of 20 dB are typical. Variation of gain with bias is such that the maximum magnitude of the local oscillations corresponds to the minimum conversion gain (a loss of more than 30 dB). No explanation can yet be given for this unexpected phenomenon.**

INTRODUCTION

The self-oscillating tunnel-diode mixer, which is attractive because it does not require an external local oscillator, has been suggested by a number of authors [1]–[3].

The inherent difficulty of a self-oscillating tunnel-diode mixer lies in the fact that since the diode does not provide isolation between the input, output, and oscillator circuits, it is difficult to independently control the frequency of oscillation [4]. The depletion-layer capacitance has to be resonated at two frequencies (signal and pump) which are close to each other and, at the same time, one must ensure that proper couplings of the respective impedances to the diode are achieved together with the stability requirements. The handling of the diode at the intermediate frequency is a minor problem. In the case of an externally applied local oscillator, the coupled impedance of the local oscillator circuit does not come into the problem since this impedance does not have a role in the principle of operation.

In the following sections, an experimental self-oscillating tunnel-diode mixer operating at the signal frequency of 600 MHz is described. A rejection-type tuning filter is used to resonate the tunnel-diode depletion-layer capacitance at the signal and self-oscillation (570-MHz) frequencies, which provides a means of independently controlling the self-oscillation frequency.

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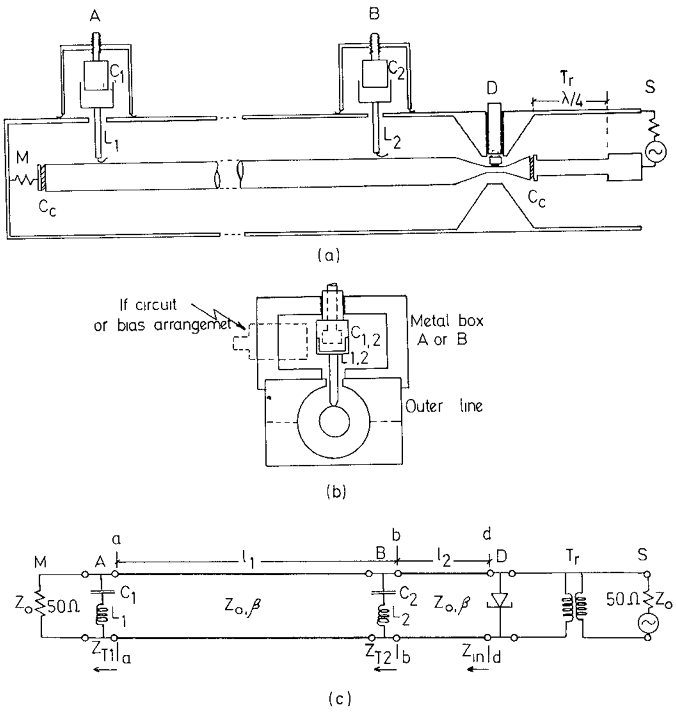

Fig. 1. (a) Schematic representation of the rejection-type tuning filter. (b) Cross section of the metal box and the line. (c) Electrical equivalent.

Rejection-Type Tuning Filter for Signal and Self-Oscillation Circuits

Fig. 1(a) shows the schematic representation of the rejection-type tuning filter for the signal and self-oscillation circuits. A and B are lumped-element series tuning circuits which are housed in rectangular metal boxes making sliding contact with the rectangular-shaped outer of the line. The inductors make sliding contact with the inner conductor of a 50-Ω coaxial line. Whole unit A or B can be moved along the line to tune out the diode capacitance in the manner described below. The diode is connected in shunt with the line at point D. This portion of the line is tapered down, keeping the 50-Ω characteristic impedance constant to the size of the tunnel diode in order to keep the lead inductance to a minimum. From D to the source S is a quarter-wave impedance transformer T_r, which presents a suitable impedance to the diode for oscillations and for different gain conditions. Fig. 1(b) shows the cross section of the box A or B and the line.

To ensure that no spurious oscillation exists outside the frequency bands employed, a matched load M terminates the rejection filter. This matched termination ($R_N > Z_0$ where R_N is the diode negative resistance) is not effective at the three frequency bands of interest, but anywhere outside these frequency bands it loads the tunnel diode and provides a means of stabilization. This is a method currently used to obtain stabilization in reflection-type tunnel-diode amplifiers [5].

Fig. 1(c) is the electrical equivalent of the setup shown in Fig. 1(a). (For a complete electrical circuit, see Fig. 2.) In this circuit, the line lengths l_1 and l_2 and the capacitance values C_1 and C_2 are all variable. The positions of A and B are evaluated as follows.

1) At the self-oscillation frequency $\omega_2 = 2\pi f_2$, $\omega_2 L_2 = 1/\omega_2 C_2$. This gives $Z_{T2}(\omega_2) = 0$. Then l_2 is obtained as

$$l_2 = \frac{\lambda_2}{2} \tan^{-1} (1/Z_0 \omega_2 C_d) \tag{1}$$

where λ_2 is the wavelength at f_2, Z_0 is the characteristic impedance, and C_d is the diode capacitance. Since no power propagates to the left of L_2C_2 at this frequency for a given value of C_d, whatever the value of L_1C_1 is, l_2 independently controls the frequency of the self-oscillations.

2) At the signal frequency $\omega_1 = 2\pi f_1$, $\omega_1 L_1 = 1/\omega_1 C_1$. This results in $Z_{T1}(\omega_1) = 0$. Again, no power travels toward M. After a detailed analysis one obtains l_1 as

$$l_1 = \frac{\lambda_1}{2} \cot^{-1} \left(\cot \beta_1 x - \frac{Z_0}{X_2} \right), \qquad \beta_1 = \frac{2\pi}{\lambda_1} \tag{2}$$

where

$$x = \frac{\lambda_1}{2\pi} \tan^{-1} (1/Z_0 \omega_1 C_d) - l_2 \tag{3}$$

and $X_2 = (\omega_1 L_2 - (1/\omega_1 C_2))$ is the reactance of the self-oscillation circuit at the signal frequency. l_1 is a continuous and single-valued function of x in the region $0 < l_1 < \lambda_1/2$. In other words, a point in the interval $0 < l_1 < \lambda_1/2$ exists such that when short circuited, the diode capacitance undergoes another resonance. Therefore, to obtain the position of the signal-series-circuit L_1C_1, x is first obtained from (3) and is then substituted in (2).

For typical circuit parameters $Z_0 = 50$ Ω, $C_d = 2$ pF, $C_2 = 5$ pF, $f_2 = 570$ MHz, and $f_1 = 600$ MHz; (5), (10), and (12) yield $l_2 = 10.6$ cm, $x = -0.64$ cm, and $l_1 = 22.3$ cm, respectively. These values are very near the experimentally observed results.

Fig. 2 shows the schematic diagram of the rejection-type tuning filter together with the intermediate (output) frequency circuit and the bias arrangements. The loading effect of the signal circuit and the matched load M are prevented by the coupling capacitances C_c (20 pF).

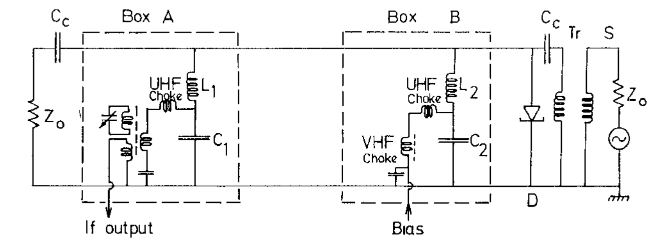

Fig. 2. Schematic diagram of the intermediate frequency and bias arrangement as incorporated in box A and B, respectively.

Growth of Oscillations in the Presence of Two Resonators

In order to show, under the conditions of the setup shown in Fig. 2, that the tunnel diode will oscillate at the pump frequency f_2, and not at the signal frequency f_1, the findings of Reich [6] can be adopted. Under certain conditions, Reich concludes that the frequency of the steady-state oscillations are govened by the tuned circuit having the smallest shunt conductance, if there is not much difference in the capacitances of the tuned circuits. The conditions under which the above conclusion are valid are readily applicable to the tunnel-diode case provided that the tunnel-diode $I-V$ characteristics are given by

$$i = a_1 v + a_3 v^3 \tag{4}$$

at the bias point $i = 0$, $v = 0$.

This is a crude representation of the tunnel diode, but for small excursions from the bias point it gives a good approximation. a_1 and a_2 are evaluated from

$$\frac{d_i}{d_v} = a_1 + 3a_3 v^2 = G(v) \tag{5}$$

where $G(v)$ is the conductance of the tunnel diode at $v = 0$ and the peak current point corresponds to

$$\frac{d_i}{d_v} = 0. \tag{6}$$

Owing to the losses of the extra length of line l_2 plus some component of the self-oscillation circuit loses, the signal-loss conductance across the diode is greater than that of the self-oscillation circuit. Therefore, the tunnel diode will oscillate at f_2.

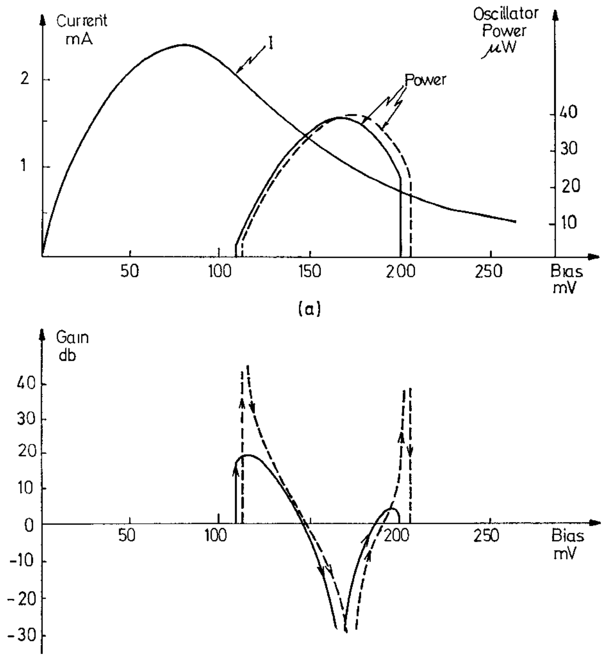

Fig. 3. (a) *I–V* characteristics of Microwave Associates tunnel-diode MA4605B. Superimposed on it is the self-oscillation power. (b) Variation of conversion gain with applied bias. Solid lines represent the result when maximum gain is adjusted to 20 dB; broken lines represent the results when the mixer is adjusted to give oscillations.

Experimental Results

Fig. 3(a) shows the diode *I–V* characteristics of Microwave Associates type MA4605B, and superimposed on it is the self-oscillation power dissipated in the source resistance (50 Ω). Fig. 3(b) shows the variations of gain with respect to bias; the effect of the voltage-dependent depletion-layer capacitance on the self-oscillation frequency is taken into account by retuning at each applied bias.

As will be seen, the maximum oscillation power region corresponds to the minimum conversion gain (a loss of more than 30 dB). This is also observable in the broken line case where the bias point for both the maximum oscillation power and the minimum gain positions shift slightly. The experiments are repeated with two other tunnel diodes differing in characteristics and exactly the same conditions are obtained. The positive gain regions in the two different bias regions correspond more or less to the same oscillation power level.

The difference in gains for small and large oscillator magnitudes has been calculated by Barber [7]. Large critical gain with low self-oscillation power coincides with Barber's results; however, the high losses encountered in the case of self-oscillations having large magnitude cannot be explained by Barber's results. The origin of this anomalous behavior is not yet understood.

The image rejection of the mixer was about 15 dB independent of all gain conditions, and the noise figure for positive gains was 6 dB. Fig. 4 shows the linearity of the self-oscillating tunnel-diode mixer for different gain conditions.

Conclusions

In view of the techniques used, it is feasible to extend the operating frequencies into the higher microwave region.

The measured noise figure of the mixer is higher than those reported previously. This is partly due to the sliding contact resistances and the losses occuring in the lumped elements, and is partly due to the source impedance which is not optimized. The latter is not curable because the source impedance has to be of the order of the negative resistance of the tunnel diode at the operating bias to cause oscillation.

The sharp and well-defined dependency of the minimum gain on the peak of the oscillation magnitude suggests that some correlation

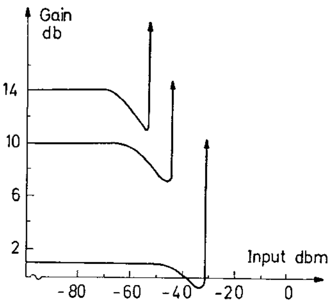

Fig. 4. Conversion gain as a function of input signal power. Steep lines with the arrow heads represent oscillations.

exists between the maximum oscillation magnitude and the available conversion gain. An explanation without a proof may be that at this bias point, all the available RF power from the tunnel diode occurs at the oscillation frequency and, consequently, the power output at any other frequency is largely minimized.

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P-I-N Variable Attenuator with Low Phase Shift

W. J. PARRIS

***Abstract*—A broad-band MIC current-controlled variable attenuator that exhibits small phase change versus attenuation has been developed. Design factors that effect phase shift are discussed. Performance at frequencies between 0.5 and 3.0 GHz is shown.**

Measured values of attenuation are shown to be independent of frequency from 0.5 to 3.0 GHz.

The development of solid-state radar systems using electronically steerable array-type antennas has generated the need for electronically variable microwave-integrated-circuit attenuators that display minimum phase change versus attenuation. These attenuators must also be impedance matched at both the input and the output terminals. This short paper describes the problems encountered and the solutions devised to produce a minimum phase change versus attenuation impedance-matched electronically variable attenuator.

To meet the requirement that the input and output impedances remain constant with changing attenuation, it is necessary to provide three variable elements in a passive attenuator. This is readily realized with the conventional tee- or pi-section resistive attenuator. A handy form of the variable resistor at microwave frequencies is the p-i-n diode. For the present project, silicon p-i-n diodes[1] in chip form were selected and evaluated in terms of microwave impedance versus forward current.

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[1] Type DSC-6157-99 from Alpha Industries, Woburn, Mass.